# Holly Park School Calculation Policy 

Multiplication and Division

## EYFS

## Early Learning Goal:

## Children solve problems including doubling, halving and sharing.

The focus in EYFS is on developing children's understanding and use of associated vocabulary linked to doubling, halving and sharing. Children need practical experience, linked to real life and engaging contexts e.g. sharing food at a picnic, birthday parties, stories etc. Sharing food for The Very Hungry Caterpillar(s) is one example of this:


The activity 'Pirate Panda' from Nrich is another good example of this, in which children are asked to reason about what is fair sharing.

## Doubling and Halving

From the beginning, children need to see the link between doubling and halving and that halving undoes doubling. So, if they know double 3 is 6 , they also know that half of 6 is 3 . Using multilink can support this concept:


The animated series 'Numberblocks' is also useful for teaching doubling (Series 2 Episode 9). There are NCETM materials for each episode to support teaching:


Children need to be able to spot equal and unequal groups and have experience of discussing whether groups are equal or not.

Have the grapes been shared equally?

Children do not use formal notation i.e. multiplication or division symbols.

Key vocabulary:double, halve, share, equal, the same amount as, not the same amount as

## Year 1

NC Objective: Solve one step problems involving multiplication and division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Non statutory guidance: Children make connections between arrays and number patterns and counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s .

Children need frequent, repeated experience of counting in 2 s , 5 s and 10 s . They need to experience oral counting as a class forwards, backwards and from different starting points, using number lines, counting sticks and hundred squares. It is useful to record the class doing this on an iPad and then play it back to them. Counting should be part of lots of Maths Skills sessions.


As well as this ordinal counting, children need lots of experience of whole class cardinal counting to count collections of objects, beginning with things that are naturally paired or in 5 s , such as pairs of socks and fingers, Numicon 2s, 5 s and 10s, plastic crabs (10s) and starfish (5s). . This will help them begin to unitise, i.e. to make the shift from being able to see 2
as ' $a 2$ ' and 5 as ' $a$ '. Children will have experienced unitising in their work on place value, thinking about 'a ten' and 'tens' rather than just ten ones. The book 'One is a snail, ten is a crab' is very useful here -

each animal is a unit!
Children need to make links with repeated addition and to understand that we can represent, for example, 5 crabs both as $10+10+10+10$ $+10=50$ and as 5 lots/groups of 10 makes 50 altogether.

Children use pictures of things that are naturally paired or in 5 s or 10s to count totals as well. They need to develop the ability to see a collection of objects in two different ways - eg ' 10 shoes' is also '5 pairs of shoes'. This ability is essential for multiplication.


Next, children count collections of things which are not naturally paired or in 5 s or 10 s but are grouped this way. Again they need both concrete and pictorial representations here. Then, they count collections of things which are not already grouped - they group objects themselves or draw rings around pictorial groups.


Wrapping paper is a useful source of images for grouping in this way.

## Part-whole models

Children use cherry diagrams to model equal parts making up a whole.


Once children have had experience of unitising, the next step is to use $2 p, 5 p$ and 10 p coins. For children to do this, they need to be able to understand that coins have value unrelated to their size, colour or shape. This is quite a challenging and abstract idea.

## Arrays

Children need to see the link between equal groups and the rows within an array. This can be done by moving counters arranged in equal groups so that each group becomes a row in the array, as shown in the three pictures below.


Children need practical exploration of arrays used in everyday life, including those in which they can collect/ group objects e.g. egg boxes, chocolate trays.


Children can make their own arrays with objects e.g. natural materials. They also need to be exposed to what is notan array:


Children can link step counting to arrays by counting down one row at a time.

## Doubling and Halving

Children need to be secure with doubles and halves number facts up to double $10 /$ half 20 . They need to understand the link between them; i.e. that halving 'undoes' doubling. ("Double 6 is 12 , so half of 12 is 6 ") As in the EYFS, multilink are very useful to support the development of this concept. These are very important and useful number facts as doubling and halving underpin a lot of later mental calculations, and are also concepts which will support children with the idea of scaling later on.

Division


Children will become familiar with the concept of division through sharing and grouping concrete objects equally. Division is complex because it has two quite different structures which essentially reverse each other; sharing and grouping. At this stage children use the language 'shared between' and 'grouped into' to represent the structures. Children solve practical problems with concrete objects and pictorial representations.

Concrete and pictorial representations include cherry diagrams (part-whole models) like these:


Please note that there is no National Curriculum requirement to use formal notation (multiplication and division symbols) at this point, White Rose Maths does use them; however we have chosen not to and instead focus on key concepts, vocabulary and number facts. We don't say that 'multiplication always makes things bigger' as later children will discover that this is not the case when we multiply by 0,1 or a fraction.

Key vocabulary: Groups, repeated addition, groups of, lots of, times, array, columns, rows, share, share equally, one each, two each, grouped into, doubled, halved.

## Year 2

## NC Objectives:

Count forwards and backwards in 2s, 3s and 5s from any given number

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals (=) signs
- Show that multiplication of 2 numbers can be done in any order (commutative) and division of 1 number by another cannot
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

Children revisit equal and unequal groups, ensuring that they are secure with the concept of equality and are able to identify such groups:

Are the groups equal?


Children begin to use the symbol for multiplication. We use the stem sentence:

There are _ equal groups with _in each group.

This is first recorded as repeated addition. Then the link is made to the multiplication symbol which we read as 'lots of' or 'groups of', before introducing the word 'times'. We don't use the language 'multiplied by' at this stage as it is not interchangeable with 'lots of' and 'groups of'.

Children continue to practise oral and object counting in steps of 2, 5 and 10, backwards and forwards from different numbers. They now move on from this to understanding and recalling the 2, 5 and 10 times tables and use the symbols and vocabulary associated with this. They also learn to step count in 3s, forward and backwards from different numbers.


'How many wheels are there? Count in groups of three.'


In Year 3 at Holly Park - children recap the 2, 5, 10, times tables
Shifting from step counting to understanding and recalling tables facts both in and out of order can be tricky. Steps in this process can be broken down as follows:

1. Confident, fluent step counting forward and back from different multiples of number in question, eg counting back in 5 s from 45, or forwards in 2s from 12.
2. Practising times tables orally in order, eg "one 5 is 5 , two 5s are 10, three 5 s are $15 \ldots$..."
3. Recalling these facts by going through the table or step count, eg counting in 5 s on your fingers to find three 5 s, or stating tables facts up to this point.
4. Recalling these facts when not in order, eg being able to answer "what is 3 times 5?" quickly and easily.

One way of helping children with this process is to use flashcards which have the multiple (eg 15) on one side and the factors (eg $3 x$ 5 ) on the other side. Children start by putting the multiples in order in a column and use this to support their recounting of the table ("one five is five, two fives are ten..."). Next they turn over the cards they are secure with and practise the others, moving finally to being able to state all tables facts in order. Once this is secure, children can practise tables in random order by placing cards with either the multiple or the factors upwards and stating what is on the other side.


Children need lots of practise to learn their times tables. Models and images to represent these tables so that they understand what they are learning include: Number lines, counting sticks, clock faces, coins, Numicon, plastic crabs, starfish etc. Counting sticks are particularly useful in helping children to see the link between step counting and equal groups. Games during Maths Skills sessions will support this learning too, including Times Table Bingo and simple card games.

Children also revisit arrays. As in Y 1 , it is useful to remind children that each row/column in an array represents an equal group, and this can be done by making equal groups and then moving them into an array one row/column at a time.


Arrays are very useful in teaching children that multiplication is commutative - ie that because $3 \times 5=15,5 \times 3=15$. This can be clearly seen in an array and it is helpful to draw rings around both sets of equal groups; the rows and the columns, in order for children to see this.


Children also start to use bar models to represent multiplication, again using the language of parts and wholes. They can do this initially using concrete objects and can move equal groups into the parts and then the whole to see the equivalence.


The distinction between the two representations - arrays and bar models - can be confusing for children. These models represent the relation between the parts and the whole differently; in bar models the whole is represented alongside the parts; ie the total amount is actually represented twice; in arrays the whole is the total of all the parts (the rows/columns). Bar models show us, for example, 3 groups of 4 but this is not identical to 4 groups of 3 . Arrays show us that 3 groups of 4 is equal to 4 groups of 3 .

Please note that we don't say that 'multiplication always makes things
bigger' as this is not the case when we multiply by 0,1 or a fraction.

## Division:

Division can be difficult and one of the key reasons for this is that it has two main structures which essentially reverse each other; sharing and grouping, With sharing, we know how many groups we need and are trying to find out the number in each group. An example of a sharing problem might be:

There are 20 pencils. If they are shared equally between 5 children, how many pencils will each child have?


We know there are 5 children, so there will be 5 groups of pencils, and we are trying to find out how many pencils each child (group) will have.

With grouping, we know how many in each group and are trying to find out how many groups there are. An example of a grouping problem might be:

There are 20 pencils. We want to give some children 5 pencils each. How many children will be able to have 5 pencils?


This time, we know how many pencils in each group - 5 - and we are trying to find out how many groups (children) there will be.

Both problems can be represented by the same number sentence $20 \div$ $5=4$. It is important that the distinction between sharing and grouping is made clear to children by talking about this and by modelling with concrete objects both sharing one at a time and grouping. While children's first experience of division is through sharing, it is grouping which most closely links to multiplication as the inverse.

Sharing: Children begin by sharing equally, using concrete resources such as multilink. They can explore which numbers can be shared into equal groups and record this using pictures, partpart whole models including cherry diagrams and bar models.

Grouping: children begin by grouping practically using objects, linking to equal groups. Offer more open ended investigations e.g. How many different ways can you group 30 counters? We can also use repeated subtraction to represent grouping and use a counting stick or number line for this - it can help children to see the link with multiplication.

Division as repeated subtraction (i.e. grouping) on a number line


It is important to be aware that it is difficult to use a bar model to
represent and solve a 'grouping' problem, as when the context is a grouping one, we don't know how many groups there are! So we don't know how many equal parts to draw alongside the bar representing the whole. Of course, the answer would be the same if we represent a grouping problem with a 'sharing' bar model, but it is confusing for the children. Instead, at this stage use bar models to represent grouping problems once they have been solved. These bar models represent the two pencils problems above:

## 20 pencils shared between <br> 5 children:



Help children to make the link between sharing and grouping e.g. What is different about sharing into two groups and grouping in twos? Children can match different bar models to word problems.

Children apply this understanding to solve problems using pictorial representations and number facts.

Arrays are also very useful to help children see the connection between division (as grouping) and multiplication. For example, a $7 \times 5$ array gives us two multiplication sentences and two division sentences:
$7 \times 5=35$ or 7 groups of 5 equals 35
$5 \times 7=35$ or 5 groups of 7 equals 35
$35 \div 5=7$ or 35 grouped into 5 s equals 7 groups
$35 \div 7=5$ or 35 grouped into 7 s equals 5 groups


Children need lots of experience working with arrays to do this. When they are secure with this they can move on to using cards with the numbers 35,5 and 7 on and finding all the possible number sentences that can be made with them.

Once children understand the link between multiplication and division, they can begin to recall the associated division facts when they practise their times tables.

Children also explore odd and even numbers through finding out which numbers can be grouped into twos.

Key vocabulary: Groups, repeated addition, groups of, lots of, times, array, columns, rows, share, share equally, one each, two each, grouped into, doubled, halved, multiple, multiply, x, divide, divided by, divided into, $\div$

## Year 3

## NC Objectives:

## Pupils should be taught to:

- Count from 0 in multiples of 4, 8, 50 and 100
- Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Step counting and tables
Children practise oral and object counting in steps of 3, 4, 8, 50 and 100, backwards and forwards from different numbers. They use number lines (both marked and empty), counting sticks, multilink, Numicon, items and images of items that are already grouped into $3 \mathrm{~s}, 4 \mathrm{~s}$ and 8 s , such as tricycles, triangles, toy animals with 4 legs like sheep, cows and toy spiders (8s).



They now move on from this to understanding and recalling the 3 , 4 and 8 times tables and use the symbols and vocabulary associated with this. As they recall these times tables they also learn the associated division facts.

In Year 3 at Holly Park - children recap the 2, 5, 10, times tables from Year 2 Children learn the 3, 4, 8 and 11 times tables

## Times tables

Shifting from step counting to understanding and recalling tables facts both in and out of order can be tricky. Steps in this process can be broken down as follows:
5. Confident, fluent step counting forward and back from different multiples of number in question, eg counting back in 4 s from 40, or forwards in 8 s from 16.
6. Practising times tables orally in order, eg "one 4 is 4 , two 4 s are 8, three 4 s are $12 \ldots$..."
7. Recalling these facts by going through the table or step count, eg counting in 4 s on your fingers to find three 4 s , or stating tables facts up to this point.
8. Recalling these facts when not in order, eg being able to answer "what is $3 \times 4$ ?" quickly and easily.

They consolidate their knowledge of tables through games like TT Rockstars, Hit the Button, Times Table Bingo etc. These are available for home learning.


One way of helping children with this process is to use flashcards which have the multiple (eg 12) on one side and the factors (eg $3 x$ 4) on the other side (see above). Children start by putting the multiples in order in a column and use this to support their recounting of the table. Next they turn over the cards they are secure with and practise the others, moving finally to being able to state all tables facts in order. Once this is secure, children can practise tables not in order by placing cards with either the multiple or the factors upwards and stating what is on the other side.

Children need lots of practise to learn their times tables. Models and images to represent these tables so that they understand what they are learning include: Number lines, counting sticks, clock faces, coins, Numicon, plastic crabs, starfish etc. Counting sticks are particularly useful in helping children to see the link between step counting and equal groups. Games during Maths Skill sessions will support this learning too, including Times Table Bingo and simple card games.


It is important for children to see the links between the 2 and 4 and the 4 and 8 times tables, i.e. that every other multiple of 2 is also a multiple of 4 , and every other multiple of 4 is also a multiple of 8 . (And every fourth multiple of 2 is a multiple of 8 ) Looking at counting sticks and number lines is a good way of making this clear, as is shading multiples of 4 and 8 on a hundred square. They can also make sticks of 2 or 4 multilink cubes and then double these (stick them together) to make 4 s or 8 s .

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| :---: | :---: | :---: | :---: | :---: | :---: |
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|  | (4) | (4) | (4) | (4) | (4) |
| 212 | 212 | 212 | $2{ }^{2}$ | 212 |  |
|  |  |  |  |  |  |



3 fours
Multilink showing the link between the 4 and 8 times tables:


Children can deepen their understanding through reasoning e.g. playing Always, Sometimes, Never. For example, are the following statements always, sometimes or never true, and why?

1. Multiples of 4 are multiples of 8 .
2. Multiples of 8 are multiples of 4 .
3. Multiples of 4 are even.

Reasoning about these will help children see the connections between the two tables. Understanding these connections also means children can use informal mental strategies to quickly calculate any forgotten times tables facts, too. If you have forgotten $8 \times 8$, but you know you can double your 4 s to find your 8 s, you can use $8 \times 4=32$ and then double 32 to get 64 .

## Calculating

Children begin to use their tables facts to multiply two digit numbers by a one digit number.

Initially, children multiply a multiple of 10 by a single digit number practically, using concrete materials, their understanding of place value and their times tables to see that, for example, because $2 \times 3=6,2 \times 30$
$=60$. It is very useful to set up 2 groups of 3 ones, and then swap the ones for tens, so that children can clearly see that multiplying tens follows the same rules as multiplying ones - just as 2 lots of 3 ones makes 6 ones, 2 lots of 3 tens makes 6/ tens. Dienes, Numicon, coins Calculation Policy
and place value counters can all be used for this but it is best to use Dienes and Numicon first so that children who struggle with unitising can really see the ten ones inside each ten.


Next, children multiply a teens number by a single digit number (which is one of the times tables they have already covered). We teach children the informal mental strategy of partitioning. Children learn to partition a 2 digit number, multiply each part of the number and then recombine by adding. Informal doubling is a very useful strategy. Children can use the doubles they know - eg to work out $2 \times 24$, you can partition the 24 into 20 and 4, double each part and recombine. While this is slightly harder once you are having to bridge past ten, this is a familiar mental addition strategy.

Arrays are a very useful way to help children see what is happening when we multiply by a one digit number greater than 2 mentally. By now children are used to arrays showing, for example, $10 \times 4$ and $3 \times$ 4. If we make an array for $13 \times 4$ we can show children why they can calculate this using the 10, 3 and 4 times tables. They need to see how the array can be split up into the part that is multiplied by 10 (10 $x 4$ ) and the other part ( $3 \times 4$ ). Examples of models and images which help with visualising this are shown below:


Dienes are also useful for children to see how we can partition numbers, operate on each part and then recombine them, as Dienes represent tens and ones already.

So, the informal method of multiplication we use is partitioning, multiplying the tens and ones separately and recombining them by adding. We also model this using an empty array or grid like this, best introduced by making a filled grid and then removing the counters/cells and writing the digits in their place:


We don't use the 'grid method' as a calculation strategy, but the grid (empty array) is useful as an image for children to understand what is happening when we partition, multiply and recombine.

In Y3, children begin to use the formal column method of short multiplication. Before they do this, they need to be confident in using the informal partitioning method which they can record in either of these ways: Calculation Policy
'One pack of biscuits costs 84 p. How much do six packs
cost?'
Example working 1:


$$
\begin{aligned}
80 \times 6 & =480 \\
4 \times 6 & =24 \\
480+24 & =504
\end{aligned}
$$

'Six packs of biscuits cost 504 p.'

Example working 2:
$84 \times 6=80 \times 6+4 \times 6$
$=480+24$
$=504$
'Six packs of biscuits cost 504 p.'

When we introduce short multiplication, children use Dienes alongside or in the algorithm. White Rose Maths represent short multiplication with place value tables alongside the column method, but we can also use these manipulatives in an expanded method, which can be clearer. Children only use place value counters once it is clear they understand their value as this is very abstract - place value counters are the same size and shape and it is not possible to see the ones inside the tens (or the tens in the hundreds).

White Rose - separate place value tables


When using manipulatives in the algorithm, it is more complex to represent a calculation for multiplication than for addition, and this is
because the numbers stand for different things (one being the number of groups, and the other being the number in the group). It is not helpful to make the 'bottom number' in the calculation a concrete representation - it is best to make the 'top number' and write the second one as a digit. You can't physically multiply one concrete item by another, and keeping them represented differently helps children to think of them as occupying different roles.

Using manipulatives in the algorithm


Children often find formal short multiplication confusing as it looks so like formal (column) addition, so it is very important that they have a good understanding of the operation before they calculate using only digits on paper and formal algorithms.

At first children multiply without exchanging/regrouping, then once they are secure with the method they move on to multiplying with regrouping. Steps are as follows:

- multiplying without regrouping
- multiplying with regrouping ones
- multiplying with regrouping tens
- multiplying with regrouping ones and tens All these steps can be modelled with Dienes and place value counters, with the exchange/regrouping being done by physically exchanging ten ones for a ten, or by crossing them off if Dienes are being represented in a drawing.

Please note that we don't say that 'multiplication always makes things bigger' as this is not the case when we multiply by 0,1 or a fraction.

## Division

In Year 3, before we teach dividing a bigger 2-digit number by a 1-digit number, we teach division by grouping with and without remainders for smaller numbers within the times tables.
For example, 8 divided by $2=4$ or 8 divided by $3=2$ R 2 .
We use blocks as the concrete resources for this and draw sticks as the pictorial representation. This is an important first step before moving onto larger numbers like 69 divided by 3.


Children continue to learn the division facts alongside their times tables, so as they learn, for example, $8 \times 9=72$, they also learn $72 \div 9=8$ and $72 \div 8=9$. As before, arrays are a very good way for children to see this link. Children need lots of experience to understand the distinction between grouping and sharing which they were first introduced to in Y2. Please see the section in Y2 titled 'Division' for a full explanation of the two structures, different representations, bar models and and why it is harder to use a bar model to solve a grouping problem.

Children are introduced to the concept of remainders in practical concrete contexts.

## Dividing a 2 digit number by a 1 digit number

As with multiplication, we teach an informal partitioning and recombining method. Children partition a number into tens and ones and then share it into equal groups, beginning with the tens and with no exchange or remainder. Children divide the tens first, and then the ones. At first they do this with Dienes and then once they are secure with using Dienes, they progress on to using place value counters.


We do not introduce formal short division in Y3.


Ron uses place value counters to solve $84 \div 2$


## Positive integer scaling problems

Scaling is a new structure of multiplication which is being introduced for the first time here. So far, children have been working on the repeated addition structure of multiplication.

Scaling is different and can be confusing. There are two different kinds of scaling with whole numbers. One involves two sets which are in a relationship with each other. For example, with the structure of multiplication we have looked at before (repeated addition), to represent 3 boxes of 10 pencils you would just draw 3 boxes, or 3 parts in a bar model. But with scaling in a problem like 'I have 10 pencils, Adelle has 3 times as many as I do' the 10 pencils I originally have don't disappear inside Adelle's - they aren't a group that is part of the product, but a separate set of pencils. As with comparison and ratio, this increases complexity. Bar models are really helpful to make this clearer.


The language is crucial here and children need lots of experience using stem sentences, following the structure "...x times as many as $y$ " and exploring what this means.. It is very helpful for children to initially encounter this in meaningful, practical contexts eg recipes, scaling up quantities of something needed. Doubling in this context is a good way to introduce the concept - we have a recipe for a cake that feeds 4 people, but we want to make it for 8 people - it is relatively easy for children to see this, particularly with concrete resources and images. This can help them to understand that if, for example, we want to make a cake that feeds 12 people, we will need 'three times as much' of everything as we do with a cake for 4.

This type of comparison structure can also be seen in measuring contexts, e.g. 'Adelle's ribbon is three times as long as Harriet's ribbon. Harriet's ribbon is 15 cm . How long is Adelle's?' The difference with the
previous example involving the comparison of sets is that here we are dealing with a continuous measure - ordinal number rather than cardinal, but again the original measure is not part of the new one that is produced in the calculation.


The other scaling structure involves an increase (if scaling by a whole number) in something, often a measure. This is different to previously encountered multiplication in that it does not involve the repetition of a set of objects (repeated addition). E.g if a sunflower grows to ten times its original height, there are not ten sunflowers but one. In this scaling structure the original measure is part of the final total, unlike in the comparison structure we have just looked at. The stem sentence we use here would be:
$\qquad$ is $\qquad$ times the length/height/mass/etc of $\qquad$
Correspondence problems are problems involving exploring all the different possible combinations that can be made by combining members of one set with members of another set, e.g. combining one set of hats with a set of coats and finding all the possibilities. The total number of possibilities can be found by multiplying the number of items in one set by the number of items in the other. Children can discover this relationship for themselves through practical investigations.


Key vocabulary: multiplication, division, array, columns, rows, share, share(d) equally, grouped into, doubled, halved, multiple, multiply, x, divide, divided by, divided into, $\div$ product, remainder, __ times as many as $\qquad$ , $\qquad$ times the length/height/mass of $\qquad$

## Year 4

National Curriculum Objectives:
Pupils should be taught to:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication and division facts for multiplication tables up to $12 \times 12$
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1 ; multiplying together three numbers
- Recognise and use factor pairs and commutativity in mental calculations
- Multiply two-digit and three-digit numbers by a onedigit number using formal written layout
- Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as $n$ objects are connected to $m$ objects.


## Step Counting and Times Tables

In Year 4 at Holly Park - children recap the 2, 5, 10, times tables from Year 2
And the 3, 4, 8 and 11 times tables from Year 3
In Year 4 they learn the 6,7.9 and 12 times tables
In Y4 children continue to practise step counting and they consolidate their knowledge of tables, learning the 6, 7 and 9 times tables and the 11 and 12 times tables. Children should know all of their times tables up to $12 \times 12$ by the end of Y4 and should also know the corresponding division facts up to $144 \div 12$. Children continue to use counting sticks and number lines, both marked and empty, Numicon and multilink. They consolidate their knowledge of tables through games like TT Rockstars, Hit the Button, Times Table Bingo etc. These are available for home learning.

When children learn the 6 and 9 times tables they look at the links with the 3 times table in the same way they did with the 2,4 and 8 times tables in Y3. They also link the 9 times table to the 10 times table.


Please look at the section on Times Tables in Y2 and Y3 for further suggestions on teaching times tables.

Children continue to use arrays and bar models throughout their work on multiplication and division.

## Multiplying by 1 and 0 , Dividing by 1

Multiplying by 1 and by 0 : Children already know their one times table in the context of other tables; they now extend this understanding to realise that 1 times anything equals itself (one group). They explore multiplying by 0 to understand that 0 times anything equals 0 (no groups). Dividing by 1 : Children understand that dividing a number by 1 equals itself. They can explore this through both sharing and grouping. Eg, if they share 24 into one equal group, all of the 24 will be in that group. Similarly, if they make equal groups of 1 , there will be 24 equal groups. Either way, $24 \div 1=24$. It is not possible to divide by 0 .

## Mental Calculations

Children use their tables and their understanding of place value to multiply mentally, e.g. knowing that because $3 \times 7=21,3 \times 70=210$ and $30 \times 7=210$. "Three lots of seven ones is twenty-one ones, so three lots of seven tens is twenty-one tens". This can be modelled using Numicon or Dienes ones which are then swapped for tens.

Children need experience in multiplying three numbers together. E.g. 2 $\times 5 \times 3=10 \times 3=30$. They are introduced to the associative law which means it doesn't matter how we group the numbers e.g. $(2 \times 5) \times 3=2$ $x(5 \times 3)$ They explore the way commutativity means these numbers can be multiplied in any order to make the same product. E.g. $5 \times 3 \times 2$ $=15 \times 2=30$, and $3 \times 2 \times 5=6 \times 5=30$. Children learn to use the
vocabulary 'factor' and 'factor pairs' and to identify factor pairs for a number using their times tables. They can then use factor pairs and commutativity to find efficient and easy ways to multiply mentally. E.g. we could calculate $3 \times 18$ by thinking of 18 as $2 \times 9$, hence $3 \times 2 \times 9$, then rearrange this into $3 \times 9 \times 2$ (as it's easiest to double as the last step).

## Formal (column method) short multiplication

Children revisit multiplication of a two digit by a single digit number using the column method, supported by Dienes and place value counters, either alongside the algorithm (as in White Rose) or as part of it (see below). They begin to multiply three digit numbers by single digit numbers in exactly the same way, using place value counters where possible at first (some children will need to continue using Dienes - see Y3 for examples of how to represent this) and progressing on to calculating without any manipulatives or other images.


When using manipulatives in the algorithm, it is more complex to represent a calculation for multiplication than for addition, and this is because the numbers stand for different things (one being the number of groups, and the other being the number in the group). It is not helpful to make the 'bottom number' in the calculation a concrete representation - it is best to make the 'top number' and write the second one as a digit. You can't physically multiply one concrete item by another, and keeping them represented differently helps children to think of them as occupying different roles.


## Formal short division

Though division is barely mentioned in the NC objectives for Y4, the nonstatutory guidance suggests that children practise formal short division (without remainders) in this year. We introduce it in Y4. Children use Dienes and place value counters to understand what is happening when they use short division. Short division is often confusing for children. The algorithm is organised completely differently from all the others - the 'bus stop' - and we write the answer (the quotient) above rather than below the calculation! Please note that short division does not appear in White Rose for Y4 but children will be expected to carry it out with numbers with up to 4 digits in Y5, so we introduce it with numbers up to 3 digits in Y4. We talk about, for example, 'how many groups of 4 tens can we make from 8 tens?' as shown below.


The NCETM CPD materials include the 'spine' for short division in Y4 which is extremely useful to look at when planning how to teach short division. In the NCETM Multiplication and Division spine, this is 'Y4 2.15 Division: Partitioning leading to short division'.

Alongside these calculations, children must continue to use pictorial representations such as bar models. These are particularly useful when solving word problems.

Please note that the purpose of manipulatives like Dienes and place value counters is to help children visualise and understand the mathematics that is going on in the calculation, particularly whenever a new method is introduced. Using manipulatives does not mean we encourage children to count in ones; instead we expect children to use their knowledge of times tables to calculate. Our expectation is that the majority of children will soon become secure with formal methods and will no longer need the manipulatives or images of manipulatives.

## Using the distributive law to multiply two digit numbers by one digit

The distributive law means that if we multiply, say, $4 \times 11$ this is equal to splitting up the 11 into two parts (say 3 and 8 ) and multiplying each by 4 separately and then adding them back together again. So, $4 \times 11=(4 \times$
$3)+(4 \times 8)$ (brackets for teacher information not children) So, we could use this to work out $7 \times 92$ mentally. We could split the 92 up into 80 and 12 , multiply each part by 7 and then add them together like this:
$7 \times 80=560$
$7 \times 12=84$
$560+84=644$

## Integer scaling problems

These are often confusing for children. There are two different structures of scaling with whole numbers. One involves comparing two sets which are in a relationship with each other. For example, with the structure of multiplication we have looked at before (repeated addition), to represent 3 boxes of 10 pencils you would just draw 3 boxes, or 3 parts in a bar model. But with scaling in a problem like 'I have 10 pencils, Adelle has 3 times as many as I do' the 10 pencils I originally have don't disappear inside Adelle's they aren't a group that is part of the product, but a separate set of pencils. As with comparison and ratio, this increases complexity. Bar models are really helpful here to make this clearer.


The language is crucial here and children need lots of experience using stem sentences, following the structure "...x times as many as $y$ " and exploring what this means.

This type of comparison structure can also be seen in measuring contexts, e.g. 'Adelle's ribbon is three times as long as Harriet's ribbon. Harriet's ribbon is 15 cm . How long is Adelle's?' The difference with the previous example involving the comparison of sets is that here we are dealing with a continuous measure ordinal number rather than cardinal, but again the original measure is not part of the new one that is produced in the calculation.

## Harriet's ribbon

 Adelle's ribbon

The other scaling structure involves an increase (if scaling by a whole number) in something, often a measure. This is different to previously encountered multiplication in that it does not involve the repetition of a set of objects (repeated addition). E.g if a sunflower grows to ten times its original height, there are not ten sunflowers but one. In this scaling structure the original measure is part of the final total, unlike in the comparison structure we have just looked at. The stem sentence we use here would be: __ is __ times the length/height/mass/etc of $\qquad$

Children will have been introduced to these scaling problems in Y 3 but will need lots of practice to fully understand this structure. The NCETM Spine for multiplication and division Y4 2.17 Scaling: Measures and Comparison has useful suggestions for practical activities here.

## Harder correspondence problems such as n objects are connected to m objects.

Correspondence problems are another multiplication structure. The number of possibilities is the number of items in one set multiplied by the number of items in the other set. Therefore if there are 12 different ice cream flavours and 4 toppings, the number of different possibilities is $12 \times 4$. Children can explore and discover this through practical activities using manipulatives and images.

Key vocabulary: multiplication, division, array, columns, rows, share, share(d) equally, grouped into, doubled, halved, multiple, multiply, $x$, divide, divided by, divided into, $\div$ product, remainder, times as many as $\qquad$ , quotient, divisor, dividend, divisible by, factor.

## Year 5

National Curriculum Objectives:

- Pupils should be taught to: identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- Establish whether a number up to 100 is prime and recall prime numbers up to 19
- Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- Multiply and divide numbers mentally drawing upon known facts
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- Recognise and use square numbers and cube numbers, and the notation for squared (2) and cubed (3)
- Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Multiples, factors, products - definitions for children
A multipleis the number that is made when two numbers are multiplied together. E.g. 35 is a multiple of 5 and of 7 . The multiples of a given number are the numbers made by multiplying that number with
other whole numbers. E.g. the multiples of 7 are $7,14,21,28,35,42$ etc.
A factoris a whole number that can be multiplied with another number to make a third number. E.g. 5 and 7 are both factors of 35. A factor can also be thought of as a whole number that divides exactly into another number.

Afactor pair is a pair of numbers which are multiplied together to form another number. E.g. 5 and 7 is a factor pair of 35 . 'All factor pairs of a number' means all the pairs of numbers which can be multiplied together to make that number. le the factor pairs of 24 are 4 and 6, 2 and 12, 1 and 24 , and 3 and 8 .

A productis the number made by multiplying two or more numbers together. E.g. 35 is the product of 5 and 7.

Children must be familiar with these through repeated experience and be able to explain what each term means.

Prime numbers, prime factors and composite numbers
A primenumberis a number that is divisible only by itself and 1.1 is not prime. 2 is the only even prime as all other even numbers are divisible by 2 . Numbers that are not prime are called composite numbers.
Children use divisibility rules to explore prime numbers and to be able to say whether a number up to 100 is prime. NRICH has a useful article on 'Divisibility Tests' and a problem solving activity called 'Division Rules'. Children practise counting in the sequence of prime numbers up to 19:

$$
1,2,3,5,7,11,13,17,19 .
$$

A prime factoris simply a factor that is also a prime number.

## Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

Children revisit column short multiplication and extend this to multiplying numbers up to 4 digits. Some children will need to use Dienes and/or place value counters to understand this. Again, when we
use Dienes or place value counters inside the algorithm (rather than in a separate place value table alongside it) we only use them to represent the 'first' number i.e. the number up to 4 digits. We don't represent the 'second' number i.e. the 1 digit number it is being multiplied by.

Please note that the purpose of manipulatives like Dienes and place value counters is to help children visualise and understand the mathematics that is going on in the calculation, particularly whenever a new method is introduced. Using manipulatives does not mean we encourage children to count in ones; instead we expect children to use their knowledge of times tables to calculate. Our expectation is that the majority of children will soon become secure with formal methods and will no longer need the manipulatives or images of manipulatives.

Annie earns $£ 1,325$ per week.
How much would he earn in 4 weeks?


## Long multiplication

Children begin this by multiplying a 2 digit number by a 2 digit number. The best way into this, which is used by both White Rose and NCETM is via the area model of multiplication. This is essentially a grid or empty array.

Martha wants to calculate how many hours there are in
January.'
$31 \times 24$


Short multiplication and combining partial products compared with long multiplication:

$\times$| 3 | 1 |  |
| :--- | :--- | :--- |
|  | 4 |  |
| 1 | 2 | 4 |$\times$| 3 | 1 |  |
| :--- | :--- | :--- |
| 2 | 4 |  |
|  | 2 | 4 |
| 6 | 2 | 0 |
| 7 | 4 | 4 |

$\times$
31
$\times \quad 20$
$\begin{array}{r}620 \\ +\quad 144 \\ \hline 744 \\ \hline\end{array}$

This helps children to see why we write a zero in the ones column of the answer when we begin to multiply with the tens digit. Children need to understand that long multiplication is a partitioning strategy and is not mathematically different to short multiplication - they are just different ways to represent the same calculation process. Because of the scale and size of the product when we are multiplying by a two digit number, children need a model which helps them to see this and we also cannot represent it very well using Dienes and place value counters in the formal algorithm. The area model works much better here.

A common misconception/error to be aware of is that children become confused between the multiplication and addition elements of the method; particularly the addition required within the multiplication to add on anything exchanged. Children also need to add up the total of the tens and the ones and the multiple calculations involved make this algorithm quite bug-prone.


Multiply and divide numbers mentally using known facts Children are taught to first think 'can I do it mentally?'. One way of doing this is to give children a range of calculations and ask them to sort them into two groups; one they can do mentally and one that are best done using the column method. Children need to be able to look, for example at 560 and know immediately that it is divisible by 7,8 , and 70 and 80 . Similarly they should be able to look at $299 \times 6$ and use its closeness to $300 \times 6$, adjusting afterwards. Children also use their knowledge of the distributive law to partition numbers in different ways and operate on them, and they use their understanding of factors and factor pairs to make calculations easier.

## Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

Children revisit formal short division of numbers up to 3 digits, using Dienes and place value counters to represent this. They then extend this to numbers with 4 digits. Some children will continue to need to see manipulatives and images. The formal division algorithm is unique in the way that it is laid out, with the answer, or quotient being written above the rest of the calculation and this can be challenging for children.


Children explore the meanings of remainders in different contexts including using decimals and finding the answer to two decimal places. The meaning of remainders changes according to the different contexts in word problems. For example:

A school of 370 children are all going on a trip. They will be split into 6 groups. How many will be in each group?

370 divided by 6 is 61 r 4 . The 4 children will have to be distributed between the groups, so 4 groups will have an extra child (ie 62) and 2 groups will have 61 children in. We have to include the remainder in the answer because we can't leave the children behind. We can't give the answer as a decimal because we can't have part of a child in a group.
$370 \div 6=$ ? as a written calculation - there is no context here and remainder can be given as $r=4$.

There are 370 bottles of juice. They will be boxed up in groups of 6. How many boxes will be filled? In this problem the answer is 61 and we disregard the remaining 4 bottles - they don't fill a box.

What to do with a remainder is often signalled by quite small nuances in the language used in word problems - it can come down to one word - and this is particularly challenging for children with EAL. It is really useful if children can experience contexts with remainders which are concrete and meaningful for them - this helps with understanding. It is also useful to give children the opportunity to compare two word problems involving the same basic calculation but in which the remainder needs to be treated differently, as in the example given above. Bar models can also be useful here.

## Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

Children need a secure understanding of place value, particularly the multiplicative aspect of place value, i.e. that each digit is worth ten times as many as it is in the column immediately to the right, and ten times fewer than in the column immediately to the left. Use the stem sentence '___(10,100,1000) of these is worth one of these/One of these is worth _ $(10,100,1000)$ of these' etc.

It is very important that children understand that we do not just 'add' a zero when we multiply by 10 , or two or three zeros to multiply by 100 or 1000. We write the zero(s) to show that we have moved the number over on the place value chart by one, two or three places. Children also need to understand that this is only true when we multiply whole
numbers as when we multiply a decimal by 10 we do not write a zero.


Children are taught the strategy of sketching a quick place value chart and 'moving' the number across the columns. The same strategy is used for dividing by 10,100 or 1000.


## Recognise and use square numbers and cube numbers, and the notation for squared and cubed

Children can explore this practically using squared paper and Multilink to discover that square numbers are literally numbers that squares can be made from, and cube numbers are numbers that cubes can be made from. Using multilink to make a series of cubes children discover the
series of cube numbers and see the link between cubed measures (capacity) and calculations like $5 \times 5 \times 5$.


Children count using the sequences of square and cube numbers e.g. in maths skills sessions:
$1,4,9,16,25,36,49,64,81,100,121 \ldots$
$1,8,27,64,125,216 \ldots$

## Scaling by simple fractions

Children have already been introduced to scaling by whole numbers (e.g. 'three times as many as...') Now, they learn about scaling in which the scale factors are not whole numbers but are unit fractions (fractions in which the denominator is one). There are two key structures here:

The first concerns contexts in which a smaller measure is being described in terms of a larger measure (comparison). For example, a spotty ribbon measures 5 cm and a plain ribbon measures 15 cm . Children will already be familiar with using stem sentences like: 'The plain ribbon is three times the length of the spotty ribbon'. Now, they describe the spotty ribbon in terms of the plain one:
'The spotty ribbon is one third the length of the plain ribbon.'


- The plain ribbon is three times the length of the spotty ribbon.'
$5 \mathrm{~cm} \times 3=15 \mathrm{~cm}$

Describe the length of the short ribbon:

## 808080808:89

- The spotty ribbon is one-third times the length of the plain ribbon.'


The second concerns something which is being decreased, and the scale factor it is to be decreased by is a fraction of the whole. E.g, a pencil was 20 cm long when new. It is now one-quarter times its original size. How long is the pencil now?


Pencil when it was new


Pencil now

$$
20 \mathrm{~cm} \times \frac{1}{4}=5 \mathrm{~cm}
$$

$20 \mathrm{~cm} \div 4=5 \mathrm{~cm}$

- 'The pencil is now five centimetres long.'

Both are as much division structures as they are multiplication ones and this link needs to be made explicit. Children need to understand that we find $1 / x$ by dividing by $x$, i.e. to find $1 / 2$ we divide by 2 , to find $1 / 3$ we divide by 3 . Generalising this experience will lead children to understand the equivalence between multiplying by a fraction and dividing by the denominator. The fact that we are making something smaller by multiplying shows us why it is important that earlier in KS1 and 2 it matters that we do not say multiplication always makes something bigger.

Please note that scaling with unit fractions is included in the NCETM Spine for Multiplication and Division in the Y4 section-2.17.

Key vocabulary: multiplication, division, array, columns, rows, share, share(d) equally, grouped into, doubled, halved, multiple, multiply, $x$, divide, divided by, divided into, $\div$ product, remainder, __ times as many as $\qquad$ , quotient, divisor, dividend, divisible by, short multiplication, long multiplication, prime numbers, composite numbers, factors, factor pairs, prime factors, square numbers, cube numbers, scaling

## Year 6

## National Curriculum Objectives:

Pupils should be taught to:

- Multiply multi-digit numbers up to 4 digits by a twodigit whole number using the formal written method of long multiplication
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Perform mental calculations, including with mixed operations and large numbers
- Identify common factors, common multiples and prime numbers

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication Children revisit long multiplication using the area model and Dienes and place value counters alongside the algorithm. A common misconception/error to be aware of is that children become confused between the multiplication part of the method; particularly the addition also required to add on anything exchanged.

'Martha wants to calculate how many hours there are in
January.'
$31 \times 24$


Short multiplication and combining partial products compared with long multiplication:


620
$+\begin{array}{r}124 \\ \hline 744 \\ \hline\end{array}$


Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
As with long multiplication, we don't use concrete materials to introduce long division because it is difficult to represent like this. By the time children are expected to use it, they should be confident in carrying out short division. They need to understand that the steps in long division are really just the same steps they take mentally when they divide using short division. Long division can be a very bug-prone
algorithm as it depends upon children remembering to 'bring down' the next digit and it has multiple steps.

Children are taught the strategy of using number sense including rounding to estimate and carrying out multiplication calculations to check. It can also be useful, particularly for children who struggle to recall their tables, to write down the multiples of the number they need.

|  |  | 0 | 3 | 6 |
| :--- | :--- | :--- | :--- | :--- |
| 1 | 2 | 4 | 3 | 2 |
|  | - | 3 | 6 | 1 |
|  |  |  | 7 | 2 |
|  | - |  | 7 | 2 |
|  |  |  | 0 |  |

It is really useful for children to draw an arrow when they are bringing down the next digit as this helps them to keep track of where they are in the calculation.

## Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

This is exactly the same as short division when we divide by single digit number. The only difference is that we are sometimes carrying two digits over to the next digit in the dividend. Children need to ensure they have enough space in their written calculations to do this. It is important that children understand that the actual calculations they are doing are exactly the same as in formal long division - it is just recorded differently.

$$
4,945 \div 23=?
$$

| Long division | Short division |
| :---: | :---: |
| 215 | $\begin{array}{llll}0 & 2 & 1 & 5\end{array}$ |
| $2 3 \longdiv { 4 9 4 5 }$ | $2 3 \longdiv { 4 { } ^ { 4 } 9 \quad 3 4 ^ { 1 1 5 } }$ |
| $4{ }^{46} 4$ |  |
| 23 |  |
| 115 |  |
| 115 |  |
| 0 |  |

For many children this is a more straightforward strategy for division by a two digit number than long division.

## Remainders

Whether carrying out long or short division by two digit numbers, how to interpret remainders depends entirely on the context. Children explore the meanings of remainders in different contexts. The meaning of remainders changes according to the different contexts in word problems. For example:

A school of 370 children are all going on a trip. They will be split into 6 groups. How many will be in each group? 370 divided by 6 is 61 r 4 . The 4 children will have to be distributed between the groups, so 4 groups will have an extra child (ie 62) and 2 groups will have 61 children in. We have to include the remainder in the answer because we can't leave the children behind. We can't give the answer as a decimal or a fraction because we can't have part of a child in a group.
$370 \div 6=$ ? as a written calculation
There are 370 bottles of juice. They will be boxed up in groups of 6. How many boxes will be filled? In this problem the answer is 61 and we disregard the remaining 4 bottles - they don't fill a box.

What to do with a remainder is often signalled by quite small nuances in the language used in word problems - it can come down to one word - and this is particularly challenging for children with EAL.

## Mental Calculations

When presented with a multiplication or division calculation, children are taught to always think first 'can I do it mentally?' To be able to carry out mental calculations confidently, particularly multi step ones and those involving large numbers, children need a good understanding of place value, times tables and how to use their knowledge of factor pairs, commutativity and the distributive law. We also continue to teach children to use partitioning strategies and part-whole models (cherry diagrams and bar models) to multiply and divide mentally or informally.

## Prime numbers, prime factors and composite numbers

A primenumberis a number that is divisible only by itself and 1.1 is
not prime. 2 is the only even prime as all other even numbers are divisible by 2 . Numbers that are not prime are called composite numbers.
Children use divisibility rules to explore prime numbers and to be able to identify larger prime numbers. NRICH has a useful article on 'Divisibility Tests' and a problem solving activity called 'Division Rules'. Children practise counting in the sequence of prime numbers up to 19:

$$
1,2,3,5,7,11,13,17,19
$$

A prime factoris simply a factor that is also a prime number.

## Using manipulatives

Please note that the purpose of manipulatives like Dienes and place value counters is to help children visualise and understand the mathematics that is going on in the calculation, particularly when a new method is introduced. Using manipulatives does not mean we encourage children to count in ones; instead we expect children to use their knowledge of times tables to calculate. Our expectation is that the majority of children will soon become secure with formal methods and will no longer need the manipulatives or images of manipulatives.

Key vocabulary: multiple, multiply, x, divide, divided by, divided into, ,product, remainder, $\qquad$ times as many as $\qquad$ , quotient, divisor, dividend, divisible by, short multiplication, long multiplication, short division, long division,prime numbers, composite numbers, factors, factor pairs, prime factors, square numbers, cube numbers, scaling

## Document Control

This policy will be reviewed biannually. Responsibility is delegated to the Governors Teaching \& learning Committee.
Revision History

| Version | Revision Date | Revised By | Revision |
| :--- | :--- | :--- | :--- |
| 1.0 | Summer 2023 | Ann Pelham \& Maths <br> Lead \& staff | Policy written |
| 1.1 | Spring 2024 | T\&L committee | Amended, Adopted \& ratified |

Signed by

|  | Name | Signature | Date |
| :--- | :--- | :--- | :--- |
| Headteacher | Ann Pelham | Delham |  |


| Chair of Governors | Clare Hegarty | chcy |  |
| :--- | :--- | :--- | :--- |

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