Progression in Addition and Subtraction

HOLLY PARK
PRIMARY SCHOOL

Contents

Holly Park Calculation Policy: Addition and Subtraction

Aims

The National Curriculum for Mathematics aims to ensure that all pupils:

- *become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.*

At Holly Park we will therefore move our children beyond simple memorisation of facts and rules and ensure they have a deep-rooted understanding of the different branches of Mathematics, and how they connect together. For our children to become fluent, they need to understand the meaning of addition and its inverse relationship with subtraction; know by heart a variety of number facts such as number bonds to 1, 10 and 100, and the commutativity of these; and a deep understanding of our place value system, how the numbers are structured within it and how they behave in addition.

The National Curriculum for Mathematics aims to ensure that all pupils:

- ***reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language.*

At Holly Park we will consider carefully what we want our children to think, notice and understand about the mathematics involved in the learning activities. We will help our children to get underneath what is going on, to make links, and to generalise their understanding. Mathematical talk will play a big part in our lessons and we will use a range of vocabulary.

The National Curriculum for Mathematics aims to ensure that all pupils:

- *can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.*

At Holly Park we will incorporate a wide range of investigations and problem solving activities into our lessons to enable children to think mathematically. We will emphasise the importance of being stuck, having another go and trying different approaches.

At every stage of calculation, we need to switch between the **concrete**, **pictorial** and **abstract** (CPA) as appropriate.

Concrete – real life objects, practical resources
Pictorial – drawing pictures of practical resources, bar models
Abstract – number lines, equations with numbers and symbols

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Vocabulary

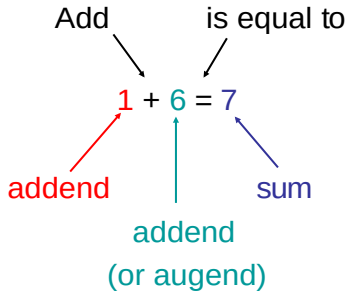
"Mathematical language is crucial to children's development of thinking. If children don't have the vocabulary to talk about division, or perimeters, or numerical difference, they cannot make progress in understanding these areas of mathematical knowledge."
Mathematical Vocabulary, DfE 2000

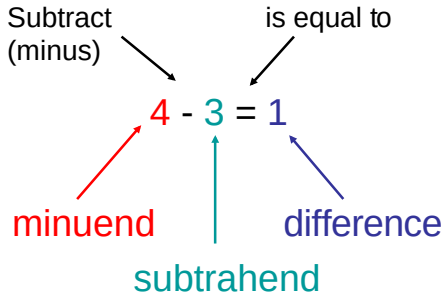
The National Curriculum for Mathematics is very clear that the correct use of mathematical language is central to a meaningful and deep understanding. Having a wide vocabulary of mathematical terminology available is essential for mathematical thinking and reasoning – we think in the same words that we speak. It is not enough for children to simply hear mathematical words; they need to 'feel' them in their own mouths. Therefore when introducing new vocabulary, everyone needs to repeat it out loud. It is also essential that new vocabulary is explained carefully and introduced alongside relevant real life contexts, practical resources or pictures so that children really understand.

Teachers need to have high expectations and only accept what is correct.

✓	×
ones	units
is equal to	equals
zero	oh (the letter O)

MATHEMATICS MASTERY

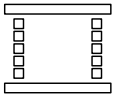




Note: 'take away' is just one strategy for subtracting

Key Vocabulary
Equality, inequality, inverse
Addition, add, plus, make, altogether
Total, sum, increase, combine
Subtraction, subtract, minus
Difference, take away, leave, decrease
More than, less than, X more, x less
Digit, tens, ones,
(near) multiple of 10
Partition, regroup, exchange

An **equation** is a mathematical statement, in symbols, that says two things are equivalent or the same [number sentence] e.g. 2=10-8, 9-3=6, 5=5



An **inequality** is a mathematical statement that two things are not the same
e.g. 3 < 10 - 9, 12 – 8 > 2 5>2

2<5

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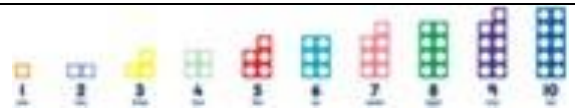
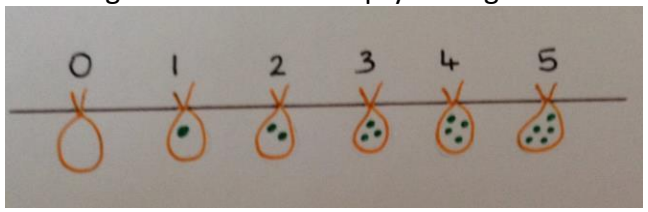
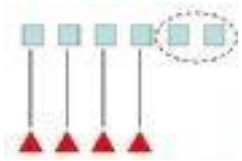
Progression in the Early Stages (mainly EYFS and KS1)

Strategy	Notes	Representations
Making groups and numbers	Recite and count the number words e.g. “two” Read and write the numbers e.g. 2	 Make a one-to-one correspondence between two groups of elements e.g. Meaning of zero 0 as an empty set e.g.  
Subitise	In Early Years, count small numbers using actions. Count things that can’t be moved. Find one more/ one less using concrete resources, fingers etc.	
Ordinal numbers	Match cardinal and ordinal numbers	Represent order and position using numbers 

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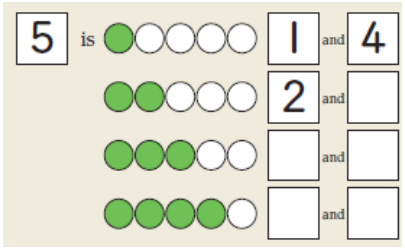
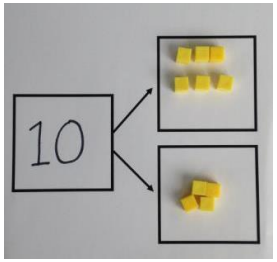
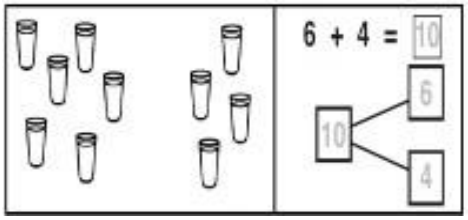
Strategy	Notes	Representations
Composing and decomposing numbers (number bonds)	Teach addition and subtraction alongside each other to show the relationship between them	Partition numbers in different ways. Consider a number as a sum or difference of two other numbers e.g.    6 + 4 =10 4 + 6 = 10 10 – 6 = 4 10 – 4 = 6
Adding together: the meaning of addition as combination	EYFS – combine 2 groups of objects together physically. Count objects from a large quantity to match a number. Encourage children to read equations aloud in different ways. Include addition that involves 0.	    Numicon shapes are great for adding without counting on – we want children to be able to ‘see’ numbers.

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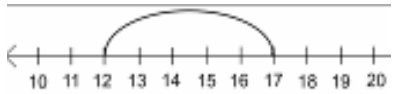
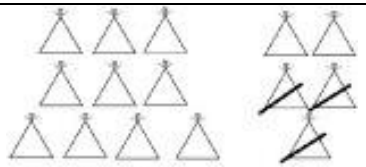
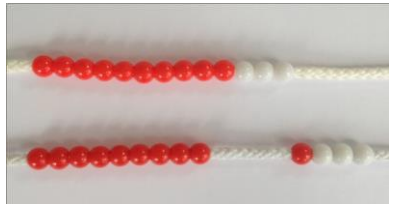
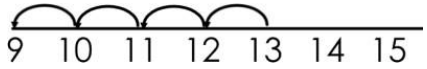
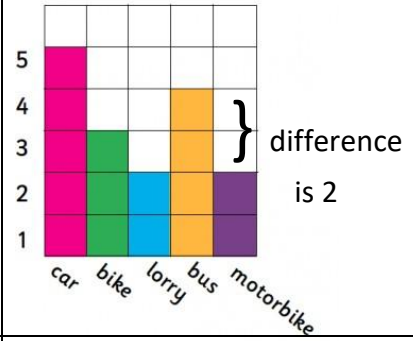
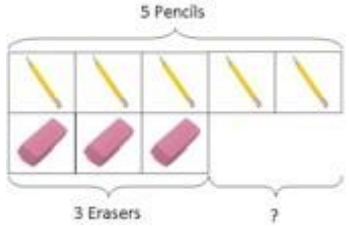
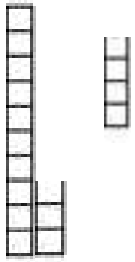
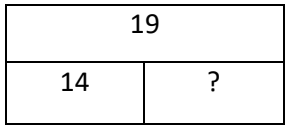
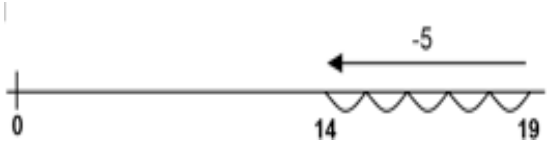
Strategy	Notes	Representations
Adding more: the meaning of addition as increase	Include addition that involves 0. Begin to count on from one group of objects to another.	Number lines can be used alongside practical apparatus to show 'counting on'. 8 + 1 = 9   17 = 12 + 5   Children can count using groups of objects.
What is left? The meaning of subtraction as decrease	When 'taking away' is first introduced, the concrete representation should be based upon the picture – place real objects on top of pictures. Include subtraction that involves 0.	  13 – 4 = 9  

Holly Park Calculation Policy: Addition and Subtraction

Strategy	Notes	Representations
What is the difference? The meaning of subtraction as difference	Record equations with the equals sign in different place. Include subtractions that involve 0.	 
Add a 1 digit number to a 2 digit number; Subtract a 1 digit number from a 2 digit number (without regrouping)	Throughout everything, solve missing number problems	13 + 4 = 17  14 = 19 - ?  

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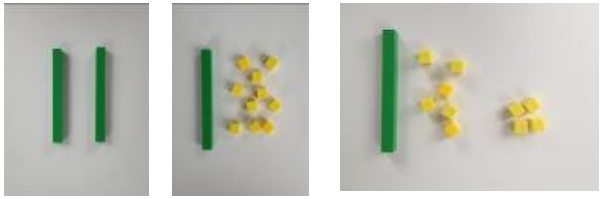
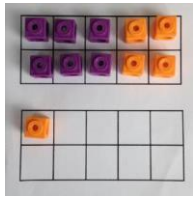
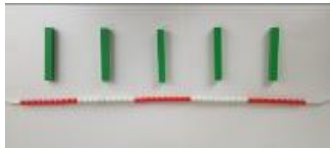
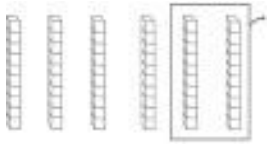
Add and subtract three 1 digit numbers (make ten first)	Children may need to try different combinations before they find the two numbers that make 10.		First: 4,7,6 (colours show >10) Second: 4,6,7 Final: total 10,7 so 17 4 + 7 + 6 = 10 + 7 10 = 17

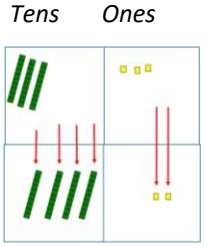
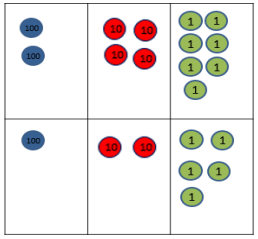
Holly Park Calculation Policy: Addition and Subtraction

Strategy	Notes	Representations
Add a 1 digit number to a 2 digit number; Subtract a 1 digit number from a 2 digit number (with regrouping)	Children may need lots of practice at physically regrouping ten ones for one ten before the two will be seen as interchangeable (e.g. use bundles of straws, dienes)	 Identify what is needed to make ten first. 20 – 4 = ? 6 + 5 = 11 
Add and subtract multiples of ten		Using the vocabulary of 1 ten, 2 tens, 3 tens alongside 10, 20, 30 is important because children need to understand that is a ten that is being added/subtracted, not a one. 30 + 20 = 50  6 tens – 2 tens =_____tens 
Add and subtract near multiples of ten by adding ten and then adjusting		e.g. subtract 9 by subtracting 10 and then adding 1 30 – 9 30 – 10 = 20 30 – 9 = 20 + 1 = 21

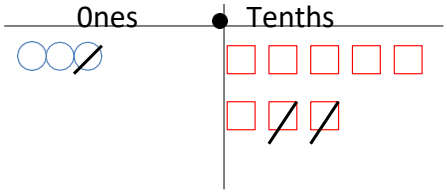
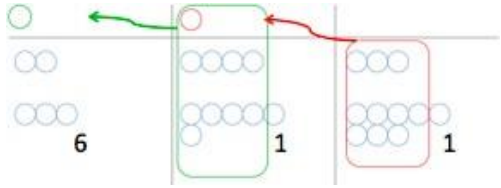
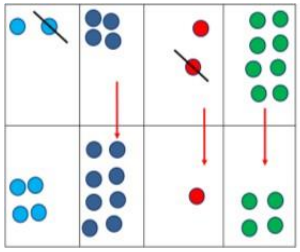
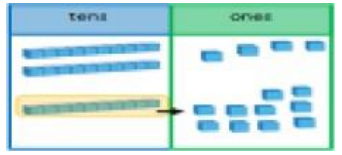
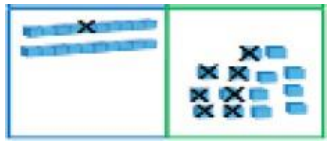
Holly Park Calculation Policy: Addition and Subtraction

Progression to Formal Methods (mainly Year 2 to Year 6)

Note: Each year the range of numbers to calculate with is extended. Every time, work through the complete sequence described below to ensure children have a deep understanding of *why* the algorithms work, not simply *how* to do them. This ensures children can apply the strategies in unfamiliar problems and increases their accuracy and reliability. For example, when teaching how to subtract decimals, start at step 1, don't just jump straight for the traditional column method and hope children make the connection with their earlier learning.

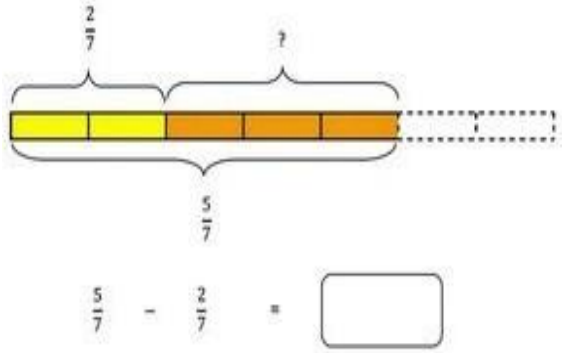
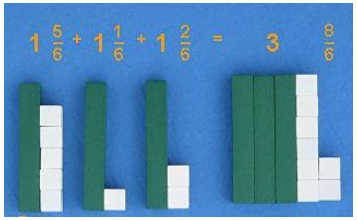
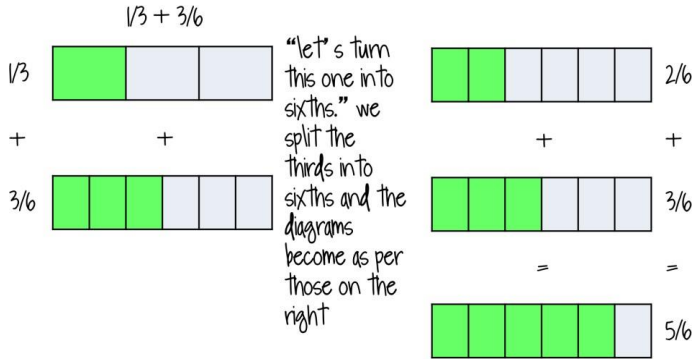
Strategy	Notes	Representations
Step 1: Left to Right Addition/ Subtraction	Place value charts and arrows can help with the partitioning e.g. 	Partition in different ways and then regroup. **47 + 25 = 60 + 12 = 72** 45 – 34 = (40 – 30) + (5 – 4) = 10 + 1 = 11 3.6 + 9.8 = (3 + 9) + (0.6 + 0.8) 12 + 1.4 = 13.4
Step 2: Expanded Written Method of Addition/ Subtraction	Start with dienes apparatus and replace with place value counters when children are confident. Then they can move to drawing the apparatus before removing them altogether.	Compare number lines and written method side by side.  70 5 -40 2 30 3  200 + 40 + 7 100 + 20 + 5 300 + 60 +12 = 372 247 +125 12 60 300 372

Holly Park Calculation Policy: Addition and Subtraction

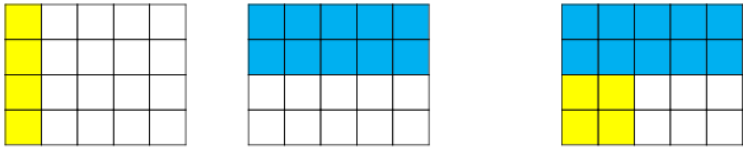
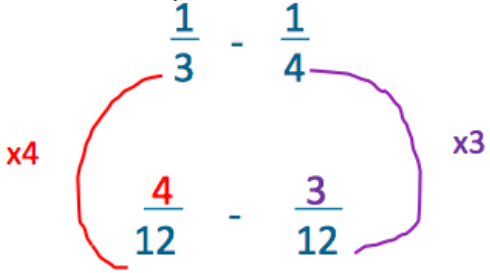
Strategy	Notes	Representations
Step 3: Vertical Addition/ Subtraction without regrouping (also called Compact Written Method)	The formal columnar method should be seen as a more streamlined version, not a new method.	3.8 – 1.2 = 2.6  7 3 3 + 2 1 2 9 4 5 The process remains the same no matter how sizeable the numbers become so encourage children to notice the pattern of what is happening and they will be able to extrapolate for much bigger numbers or decimals without hesitation.
Step 4: Vertical Addition/ Subtraction with regrouping	If you hear phrases like, "Cross out this and put this," then you need to intervene. We want conceptual understanding not memorising rules without meaning.	Use place value counters, charts and pictures to demonstrate the regrouping first: 1 1 2 4 3 +3 6 8 6 1 1   6232 - 4814 1418 34 – 17 = 17  

Holly Park Calculation Policy: Addition and Subtraction

Adding and Subtracting Fractions (mainly Year 4 to Year 6)

Strategy	Notes	Representations
Add and subtract fractions with the same denominator	Use bar models and other images to support conceptual understanding. Use Cuisenaire rods to model.	Emphasis that you can only add/subtract with the same noun e.g. 2 cars + 3 cars = 5 cars "2 one-ninths + 3 one-ninths = 5 one- ninths" You can't add 3 pears with 2 apples though! 
Write mathematical statements greater than 1 as a mixed number	Cuisenaire or number rods are great resources to use to explore the relationship between numbers.	 = 4 2/6 2/5 + 4/5 = 6/5 = 1 1/5
Add and subtract fractions with denominators that are multiples of the same number	Link to times tables – use multiplication grids to support finding of common multiples	

Holly Park Calculation Policy: Addition and Subtraction

Strategy	Notes	Representations
Add and subtract fractions with different denominators	Use bar models and other images to support conceptual understanding	Start with pictorial approach: $\frac{1}{5} + \frac{2}{4} = \frac{14}{20}$  $\frac{4}{20} + \frac{10}{20} = \frac{14}{20}$ Once the concept is understood, move onto to quicker strategies: 

Holly Park Calculation Policy: Addition and Subtraction

National Curriculum Progression: Addition and Subtraction

Taken from the NCETM

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Understanding addition and subtraction		*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)*	*Choose an appropriate strategy to solve a calculation based upon the numbers involved (recall a known fact, calculate mentally, use a jotting, written method)*
	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs	Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot *Understand subtraction as take away and difference (how many more, how many less/fewer)*	*Understand and use take away and difference for subtraction, deciding on the most efficient method for the numbers involved, irrespective of context*			
Addition and subtraction facts	Represent and use number bonds and related subtraction facts within 20	Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 *Recall and use number bonds for multiples of 5 totalling 60 (to support telling time to nearest 5 minutes)*	*Recall and use addition and subtraction facts for 100 (multiples of 5 and 10)* *Derive and use addition and subtraction facts for 100* *Derive and use addition and subtraction facts for multiples of 100 totalling 1000*	*Recall and use addition and subtraction facts for 100* *Recall and use addition and subtraction facts for multiples of 100 totalling 1000* *Derive and use addition and subtraction facts for 1 and 10 (with decimal numbers to one decimal place)*	*Recall and use addition and subtraction facts for 1 and 10 (with decimal numbers to one decimal place)* *Derive and use addition and subtraction facts for 1 (with decimal numbers to two decimal places)*	*Recall and use addition and subtraction facts for 1 (with decimal numbers to two decimal places)*
Mental methods		*Select a mental strategy appropriate for the numbers involved in the calculation*	*Select a mental strategy appropriate for the numbers involved in the calculation*	*Select a mental strategy appropriate for the numbers involved in the calculation*	*Select a mental strategy appropriate for the numbers involved in the calculation*	*Select a mental strategy appropriate for the numbers involved in the calculation*
	Add and subtract one-digit and two-digit numbers to 20, including zero *(using concrete objects and pictorial representations)*	Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and	Add and subtract numbers mentally, including: - a three-digit number and ones - a three-digit number and	*Add and subtract mentally combinations of two and three digit numbers and decimals to one decimal place*	Add and subtract numbers mentally with increasingly large numbers *and decimals to two decimal places*	Perform mental calculations, including with mixed operations and large numbers *and decimals*

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		ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers	tens - a three-digit number and hundreds			
Written methods	**Written methods are informal at this stage – see mental methods for expectation of calculations*	**Written methods are informal at this stage – see mental methods for expectation of calculations*	Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	Add and subtract numbers with up to 4 digits *and decimals with one decimal place* using the formal written methods of columnar addition and subtraction where appropriate	Add and subtract whole numbers with more than 4 digits *and decimals with two decimal places*, including using formal written methods (columnar addition and subtraction)	*Add and subtract whole numbers and decimals using formal written methods (columnar addition and subtraction)*
Estimating and checking calculations		Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems	Estimate the answer to a calculation and use inverse operations to check answers	Estimate and use inverse operations to check answers to a calculation	Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy	Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
Order of operations						Use their knowledge of the order of operations to carry out calculations involving the four operations
Solving addition and subtraction problems including those with missing numbers	Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ - 9	Solve problems with addition and subtraction *including those with missing numbers*: - using concrete objects and pictorial representations, including those involving numbers, quantities and measures - applying their increasing knowledge of mental and written methods	Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction	Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why *Solve addition and subtraction problems involving missing numbers*	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why *Solve addition and subtraction problems involving missing numbers*	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why Solve problems involving addition, subtraction, multiplication and division, *including those with missing numbers*

Holly Park Calculation Policy: Addition and Subtraction

Appendices

These appendices demonstrate possible progression through calculation strategies following the Concrete-Pictorial-Abstract (CPA) approach. The aim is always for children to calculate confidently, accurately and efficiently. The most efficient strategies are usually the formal written methods of column addition and subtraction ("abstract"). As laid out in the National Curriculum for Mathematics, using these written strategies is the expectation for children leaving Key Stage 2. It is paramount that children have a sound understanding of the concepts though, before being taught the formal methods, in order to avoid misconceptions forming. Therefore interim methods (often "concrete" and "pictorial") are taught first. The aim is always to progress from the interim methods as soon as children are confident.

The first step when teaching any calculation strategy is always to demonstrate it using concrete apparatus (e.g. Dienes). This helps to ensure conceptual understanding, not just procedural recall. Once confident with the concrete, move to the pictorial - first by drawing the apparatus alongside manipulating it and then just drawing it to as a guide. When the pictures are no longer needed, move to the abstract (the formal written procedures). It is important that the movement between each step is flexible. In other words, there needs to be opportunities for progress without blocking the way back to the sources in which understanding is grounded. For example, when children encounter a complex problem or new situation that they are uncomfortable solving solely in the abstract, they should draw a representational picture or use concrete apparatus to guide them.

Each appendix here starts with an example of how to represent the concrete Dienes apparatus pictorially and moves through other interim methods, ending with the formal written method.

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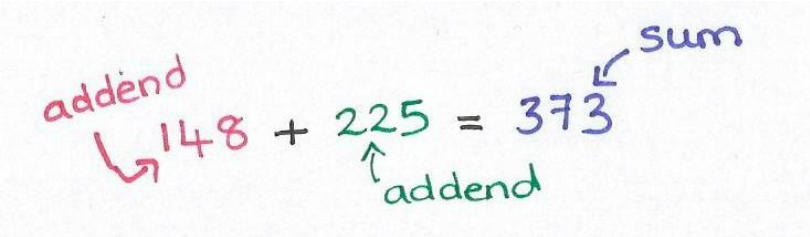
Appendix A: Building up to Written Addition



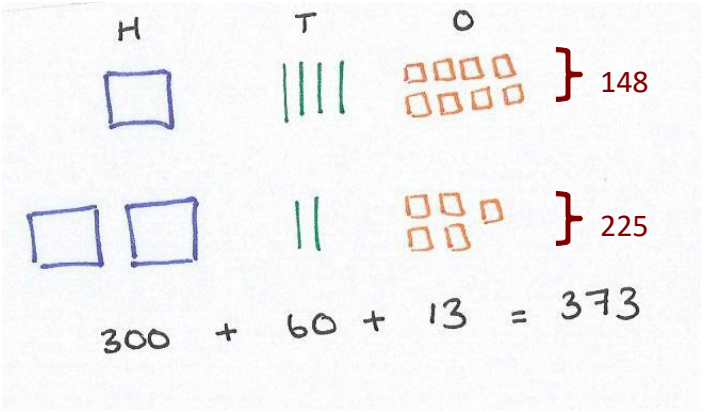
Expanded Methods

The expanded written method is an interim method before introducing the formal written algorithm because each partial sum is visible. Dienes apparatus could be placed (or drawn) alongside the numbers to show what is going on.

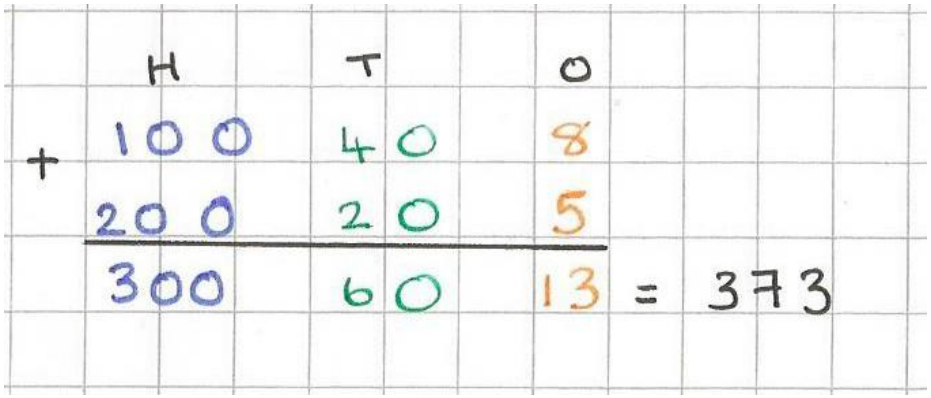
Partition (split up) each addend according to their place value e.g. 148 is made up of one hundred, four tens and eight ones.

Line up the hundreds, tens and ones of each addend and find the total of each, starting with ones and working left.

Recombine to give the final sum.



Note: the next methods use 'regrouping and exchanging'.
The expanded methods can be used to demonstrate that first.



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Holly Park Calculation Policy: Addition and Subtraction

Compact Methods

Place value counters are an excellent stepping-stone between the concrete Dienes apparatus and the abstract formal written method.

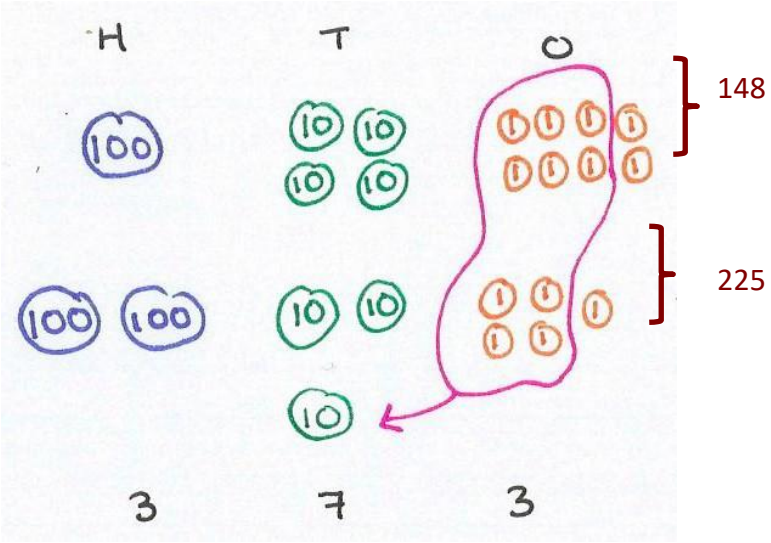
Partition (split up) each addend according to their place value and line up in columns.

We can 'regroup' the ones to make the numbers easier to work with. Bundle up ten (1) discs and exchange them for one (10) disc.

Now instead of 6 tens and 13 ones, we have 7 tens and 3 ones. The total value has not changed; we have just renamed it.

Add up how many hundreds, tens and ones there are altogether.

Recombine to give the final sum.



	H	T	O
+	1	4	8
	2	2	5
	3	7	3
		1	

Record the exchange underneath

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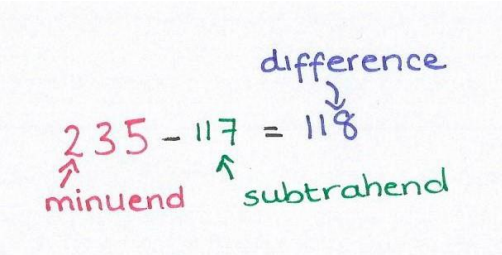
Holly Park Calculation Policy: Addition and Subtraction

Appendix B: Building up to Written Subtraction



Expanded Methods

The expanded written method is an interim method before introducing the formal written algorithm because each partial difference is visible. Dienes apparatus could be placed (or drawn) alongside the numbers to show what is going on.

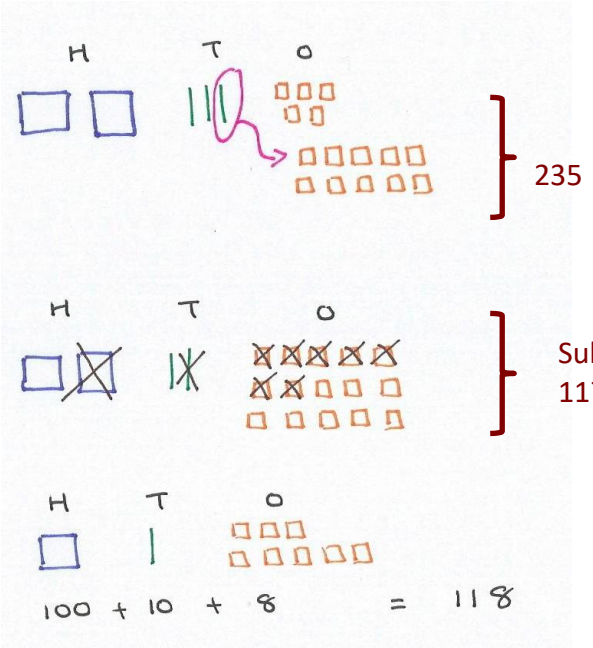
Partition the minuend according to its place value.

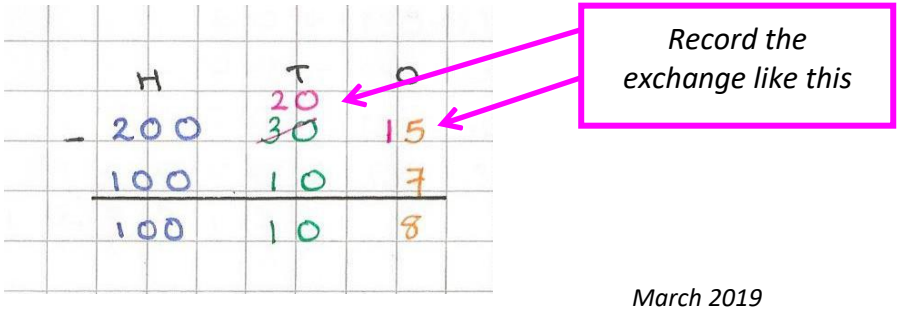
Start with the ones column and subtract (physically take away) the amount of ones in the subtrahend.
Warning! We cannot physically take 7 ones away from 5 ones. So we need to regroup. Exchange 1 ten for 10 ones.

We now have 15 ones and can easily take away 7. Note that the total value of the minuend does not change. It is always 235; we just change how it is partitioned (30 and 5 or 20 and 15).

Then subtract the amount of tens in the subtrahend from the new amount of tens in the minuend.
Likewise for the hundreds.

The amount left is the difference.





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Holly Park Calculation Policy: Addition and Subtraction

Compact Methods

Place value counters are an excellent stepping-stone between the concrete Dienes apparatus and the abstract formal written method.

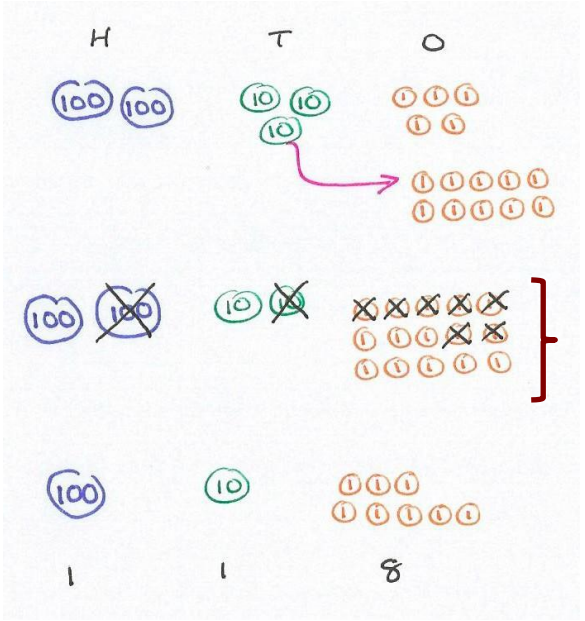
Partition the minuend according to its place value.

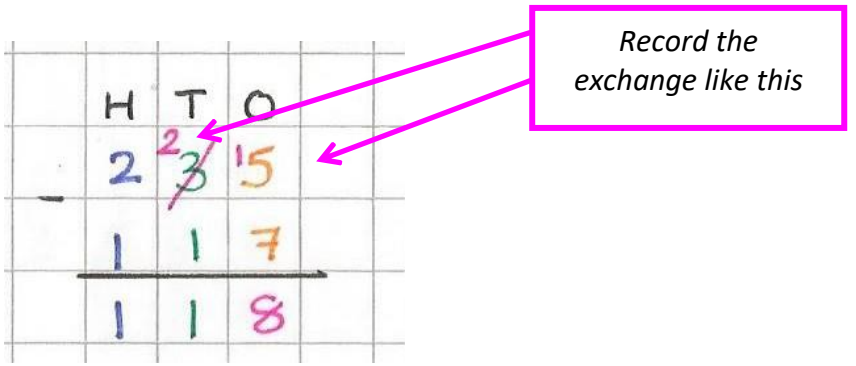
Start with the ones column and subtract (physically take away) the amount of ones in the subtrahend.
Warning! We cannot physically take 7 ones away from 5 ones.
So we need to regroup. Exchange ten (1) discs for one (10) disc.

We now have 15 ones and can easily take away 7.
Note that the total value of the minuend does not change.
It is always 235; we just change how it is partitioned
(3 tens and 5 ones or 2 tens and 15 ones).

Then subtract the amount of tens in the subtrahend from the new amount of tens in the minuend.
Likewise for the hundreds.

The amount left is the difference.





Note: in more complex examples, multiple exchanges might be required.

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